

Mortgage Financing for Muslim Americans

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Islamic economics is a rapidly growing discipline seeking to redirect economic behavior under the umbrella of Islam. This field combines malleable modern economic concepts with immutable moral principles in its attempt to address the economic climate of a given society. The most significant differences between Islamic economics and the world's more prevalent economic systems are the Islamic ban on *ribā*¹ (usury) and the Islamic institution of zakah. The ban on usury is based on the verse:

O you who believe, fear Allah and give up what remains due to you of usury if you are indeed Believers. And if you do not, then be warned of war (against you) by Allah and his messenger, while if you repent you shall have your capital. Do not wrong and you shall not be wronged (Qur'an 2:278-9).

This paper addresses mortgage financing problems on real property that Muslims face as a result of Islam's ban on interest. The United States has a unique tax system designed to encourage investment in real property. Congress allows property owners to deduct all mortgage interest, along with operating expenses and depreciation writeoffs, from income taxes. This is an indirect form of government subsidy.²

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¹El-Ashker (1987) defines *ribā* as an increase in money or kind that the debtor might be asked to pay over the principal to compensate for the payment. In modern economic terms, it could be defined as a positive risk-free return from an investment vehicle or an investment strategy. Ebrahim (1992a) explains that one may "go long": buy an index on a basket of stocks such as the Standard & Poors 500 and simultaneously sell short a futures contract against it. This results in a risk-free return and therefore constitutes *ribā*.

²Home owners can deduct only the interest expenses from their income taxes.

One modern form of Islamic mortgage financing that does not contradict Islamic values is called an equity participating mortgage.³ This type of mortgage has been studied as an alternative form of financing in the American real estate market. In addition, it should be noted that the benefits of an equity participating mortgage, as will be modelled in this research paper below, are applicable to both Muslim and non-Muslim societies.

A participating mortgage is an equity form of financing in which the lender and the borrower undertake a joint investment and agree to a future division of the profits (or losses) according to prespecified shares. Thus the lender participates in the income, the residuals, or both. The mechanism is simple: the lender loans money to the borrower in exchange for a predetermined share of the profits (or losses) earned in the enterprise plus a portion of the tax benefits for which the enterprise is eligible. Such a form of mortgage financing is unique in the field of real estate investment.

This study refers to the works of such scholars as Keeley (1987) and Levy et al. (1989), which discuss several advantages accruing to a borrower involved in a participating mortgage: a) The borrower holds the title and the ownership, and thereby retains control of the real estate; b) As both the borrower and the lender share in the profits and the losses, the borrower does not have to finance the enterprise alone and, in addition, he/she receives a cushion against bankruptcy; c) During bad economic times, one feature of which is lower debt service costs, the enterprise's poor rate of return is borne by both parties; d) The holder of the title, in this case the borrower, receives most of the applicable tax benefits; e) Higher financing proceeds, as the loan-to-value ratio is higher with equity; and f) Some of the risk is shifted to the lender.

There are also several advantages for a lender involved in such a deal: a) He/She benefits from the cash flow stream received from the enterprise and the capital gains made upon the sale of the real estate; b) Participating mortgages allow the nontaxable lender to invest in real estate and thereby add greater diversity to his/her investment portfolio; c) Real estate protects the lender's rate of return by providing a hedge against inflation (i.e., periods of inflation would be offset by an increase in the property's nominal value); and d) The lender receives some of the tax benefits associated with the enterprise.

The main purpose of this paper is to present an optimal mortgage contract that will allow nontaxable entities (i.e., pension funds) to benefit from income and appreciation opportunities in the real estate market. Such investors may be priced out of the market, since real estate investment is imbued with tax advantages and writeoffs through depreciation

³Other seller financing techniques also avoid usury: the Installment-Sale method and the Lease-with-an-Option-to-Buy method, both of which are described in Ebrahim (1991). However, these techniques are not institutionalized and depend on individual negotiation between buyer and seller.

and deductible interest payments, whereas other forms of investment (i.e., financial securities) are not. Ennis and Burik (1991b) assert that

taxes constitute an additional non-risk factor. If pension fund investment decisions take taxes into account it becomes more difficult to justify real estate investment by pension funds, regardless of fund size or plan type.

As a result, they (1991a) disagree with many scholars, among them Fogler (1984), Brueggemann et al. (1984), Zerbst and Cambon (1984), Webb and Rubens (1987), Irwin and Landa (1987), and Firstenberg et al. (1988), who advocate placing 20 percent of a pension fund's portfolio into real estate in order to provide it with diverse holdings and to enhance its return-to-risk ratio.⁴ Ennis and Burik (1991a, b) opine that pension funds have invested only 3.5 percent of their portfolio in real estate, because the Employee Retirement Income Security Act (ERISA) of 1974 admonishes pension fund investors to diversify their portfolios in different asset classes. Ebrahim (1992b) also points out that all nontaxable entities hold only 4.63 percent of American real estate for investment purposes. Thus equity participating mortgages allow nontaxable investors to circumvent entry constraints and invest in the real estate market.⁵

An optimal security design, which encompasses the allocation of cash flows and control rights, has been discussed extensively by Harris and Raviv (1991). However, their theories exclude tax considerations. Basheer (1990) has an excellent paper on profit sharing (*muḍārabah*) as an optimal contract under asymmetric information according to the Sharī'ah, but it does not include the intricacies of real estate investment under the American tax code.

This paper uses the two-period general equilibrium theory to model equity participating mortgages, as this tool is widely accepted by academics and policymakers. Risk neutrality is also used, because it readily allows the derivation of a closed-form solution. A framework akin to that of DeAngelo and Masulis (1980) is used. In addition, this study also incorporates the recapture of depreciation as capital gains. The modelling is structured according to the present tax code.⁶

⁴These scholars' results are controversial, for they have used ex-post returns based on an appraised value of real estate. It is a well-known fact that the appraisal process smoothens the returns' variance. Most of these studies, moreover, have ignored taxes. Thus the objection of Ennis and Burik (1991a, b) deserves attention.

⁵Real estate has been termed a tax shelter by such practitioners as Bruss (1990) and academics as Cordes and Galper (1985). Since there are tax benefits to investing in real property (even in the new tax law), taxable investors may outbid nontaxable investors.

⁶The Tax Reform Act of 1986 (TRA '86) reversed the incentives of the Economic Recovery Tax Act of 1981 (ERTA '81). This was due to the fact that tax depreciation was made less generous than that under ERTA '81 and capital gains tax rates were raised above pre-1978 levels. For individuals and closely held corporations, the act limited pas-

Section two provides a mathematical model for an equity participating mortgage, the solution of which shows that this mortgage expands the pool of real estate investors to include those with nontaxable income. Section three concludes the paper by placing equity participating mortgages within the framework of an Islamic socioeconomic structure.

Modeling Investment in Depreciable Real Estate under an Equity Participating Mortgage (TRA '86)

Consider the following two-period general equilibrium model. At time $t = 0$, there are N agents in the economy, who are young in period $t = 0$, old/retired in period $t = 1$, and dead in period $t = 2$ and beyond. Half of the agents/investors are in the marginal τ tax bracket, and the other half are in the zero tax bracket. This is the only form of heterogeneity in the model.

All investors have endowments of w_0 in period zero. There is no endowment at the age of retirement (time $t = 1$). The endowment w_0 can be consumed, loaned, or used to buy real estate, and is not taxed.

There are only two kinds of assets in the economy. One is a "depreciable real estate" investment that yields a stream of net operating income $\{d_1\}$ and a liquidating dividend $\{P_1\}$, where d_1 and P_1 are positive random first-order Markov processes. The second asset is a risky asset (i.e., a mortgage) in zero net supply as explained in the Market Clearing Condition in the following pages.

It is assumed that all investors are consumers of the endowment, of the real estate investment (in the form of net rent plus future price after tax), or of interest income. Investors retire in period 1 and consume whatever is left of the portfolio. The government allows a one-time tax depreciation D_0 , which is proportional to the price P_0 of the asset.

There are $(N/2)$ real estate assets in the economy, which are assumed to outlive the investors. The investors have a choice of buying real estate with a combination of endowment w_0 and a loan (the loan assumed to be between two types of investors). The interest rate i charged by a lender is the return on the loan demanded by the lender.

The analysis is done by modeling the returns to both types of investors. It then determines if the solution is an interior or a corner solution.

Modeling Objective Function of a Zero Taxable Risk Neutral Investor.

The first step is to optimize the expected utility of a zero tax investor:

$$\text{Max. } E_0\{c'_0 + \gamma c'_1\}$$

$$Q'_0, c_0, c_1, s'$$

sive activity losses to the amount of passive income. It also limited tax-deductible losses to the amount the investor had "at risk," rather than to the amount of the investor's basis in the property. Despite these restrictions, most real estate investments continue to offer some tax benefits.

Subject to:

$$(a) c'_0 + Q'_0 + s'P_0 = w_0$$

$$(b) c'_1 = Q'_0 (1 + i) + s' (d_1 + P_1), \text{ where:}$$

$E_0 \{ \}$ = expectation operator at time 0.

c'_0 = consumption of nontaxable investor at time 0.

c'_1 = consumption of nontaxable investor at time 1.

γ = discount rate.

s' = fractional investment in real estate by nontaxable investor.

Q'_0 = amount of funds lent/borrowed.⁷

P_0 = price of real estate at time 0.

w_0 = endowment at time 0.

i = interest rate.

d_1 = net operating rental income of the property received at time 1.

P_1 = value of the property at time 1.

The lagrangian L' can be written as follows:

$$L' = E_0 \{ [c'_0 + \gamma c'_1] + \lambda'_0 [w_0 - s'P_0 - Q'_0 - c'_0] + \lambda'_1 \gamma [Q'_0(1+i) + s'(d_1 + P_1) - c'_1] \}$$

The First Order Necessary Conditions (F.O.N.C.s) are given by:

$$\frac{\delta L'}{\delta c'_0} = E_0 [1 - \lambda'_0] = 0 \Rightarrow \lambda'_0 = 1 \quad (1')$$

$$\frac{\delta L'}{\delta c'_1} = E_0 \{ \gamma(1) + \lambda'_1 \gamma (-1) \} = 0 \Rightarrow \lambda'_1 = 1 \quad (2')$$

$$\frac{\delta L'}{\delta Q'_0} = E_0 \{ \lambda'_0 (-1) + \lambda'_1 \gamma (1 + i) \} \geq 0$$

Using (1') and (2'), we get:

$$\frac{\delta L'}{\delta Q'_0} = E_0 \{ -1 + \gamma (1 + i) \} \quad (3')$$

⁷A positive level of Q'_0 indicates the amount of funds lent, whereas a negative level indicates the amount borrowed.

$\geq 0, \forall Q'_0 > 0$ with equality in an interior solution and inequality in the corner solution,

$\leq 0, \forall Q'_0 < 0$ with equality in an interior solution and inequality in the corner solution.

Investors will continue to buy fractional shares in real estate until the net benefit equals zero at the margin. Similarly, investors will avoid real property if the net benefit is less than zero at the margin. This can be represented by the partial derivative of the lagrangian with reference to the fractional shares owned as given below:

$$\begin{aligned} \frac{\delta L'}{\delta s'} &= E_0\{\lambda'_0[-P_0] + \lambda'_1 \gamma [d_1 + P_1]\} \\ &= E_0\{[-P_0] + \gamma [d_1 + P_1]\} \quad (4') \\ &= 0 \text{ for an interior solution where } s' \neq 0 \\ &< 0 \text{ for a corner solution where } s' = 0. \end{aligned}$$

Modeling Objective Function of a τ Tax Bracket Risk Neutral Investor. Consider the τ tax bracket investor. The goal of this investor is:

$$\text{Max. } E_0\{c_0 + \gamma c_1\}$$

$$Q_0, c_0, c_1, s$$

Subject to:

$$(a) c_0 + sP_0 + Q_0 = w_0,$$

$$(b) c_1 = s[d_1(1 - \tau) + P_1(1 - g\tau) + \tau(D_0(1 - g) + gP_0)] + Q_0[1 + i(1 - \tau)]$$

where:

- $E_0\{ \}$ = expectation operator at time 0.
- c_0 = consumption of taxable investor at time 0.
- c_1 = consumption of taxable investor at time 1.
- γ = discount rate
- s = fractional investment in real estate by taxable investor.
- Q_0 = amount of funds lent/borrowed.⁸
- P_0 = price of real estate at time 0.
- w_0 = endowment at time 0.
- i = interest rate on Q_0 .

⁸A positive level of Q_0 indicates the amount of funds lent, whereas a negative level indicates the amount borrowed.

d_1 = net operating rental income (NOI) of the property received at time 1.

P_1 = value of the property at time 1.

τ = the marginal tax rate on interest/dividend/rental income.

$g\tau$ = effective capital gains tax levied on the appreciation of property.

τD_0 = tax depreciation allowed in the law.

k = depreciation constant (rate of depreciation allowed)

$$\text{i.e., } k = \frac{D_0}{P_0}$$

The after-tax cash flow from the operation and the sale of the property is explained in footnote 9.⁹

The lagrangian L can be written as:

$$L = E_0\{[c_0 + \gamma c_1] + \lambda_0[w_0 - sP_0 - Q_0 - c_0] + \lambda_1\gamma[s[d_1(1-\tau) + p_1(1-g\tau) + \tau(D_0(1-g) + gP_0)] + Q_0(1 + i(1-\tau)) - c_1]\}$$

The F.O.N.C.s are:

$$\frac{\delta L}{\delta c_0} = E_0[1 - \lambda_0] = 0 \Rightarrow \lambda_0 = 1, \quad (1)$$

$$\frac{\delta L}{\delta c_1} = E_0[\gamma - \lambda_1\gamma] = 0 \Rightarrow \lambda_1 = 1, \quad (2)$$

$$\frac{\delta L}{\delta Q_0} = E_0\{-\lambda_0 + \lambda_1\gamma_0[1 + i(1-\tau)]\} \geq 0$$

Substituting the value of λ_0 and λ_1 from (1) and (2), we get:

$$\frac{\delta L}{\delta Q_0} = E_0\{-1 + \gamma_0(1 + i(1 - \tau))\} \geq 0 \quad (3)$$

⁹The book value of the property at time $t = 1$ is $(P_0 - D_0) = (P_0 - kP_0)$
 The capital gains tax is therefore levied on $P_1 - (P_0 - D_0) = P_1 - (P_0 - kP_0)$
 The after-tax cash flow from the sale of the property = $P_1 - g\tau [P_1 - (P_0 - kP_0)]$
 The after-tax cash flow from operation = $d_1(1-\tau) + \tau D_0 = d_1(1-\tau) + \tau kP_0$
 \therefore Total after-tax cash flow from operation and sale of property =
 $= [d_1(1-\tau) + \tau kP_0] + [P_1 - g\tau [P_1 - (P_0 - kP_0)]]$
 $= [d_1(1-\tau) + P_1(1-g\tau) + \tau(D_0(1-g) + gP_0)]$

$\geq 0, \forall Q_0 > 0$ with equality in an interior solution and inequality in the corner solution,

$\leq 0, \forall Q_0 < 0$ with equality in an interior solution and inequality in the corner solution.

Investors will continue to buy fractional shares of real estate until the net benefit equals zero at the margin. Similarly, investors will avoid real property in the net benefit is less than zero at the margin. This can be represented by the partial derivative of the lagrangian with reference to the fractional shares owned as given below:

$$\begin{aligned} \frac{\delta L}{\delta s} &= E_0\{\lambda_0(-P_0) + \lambda_1\gamma[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g) + g)]\}, \\ &\text{where } k = \frac{D_0}{P_0} \\ &= E_0\{-P_0 + \gamma[d_1(1-\tau) + p_1(1-g\tau) + \tau P_0(k(1-g) + g)]\}, \text{ after sub-} \\ &\text{stituting the value of } \lambda_0 \text{ and } \lambda_1 \text{ from (1) and (2).} \\ &= 0 \text{ for an interior solution, where } s \neq 0, \\ &< 0 \text{ for a corner solution, where } s = 0 \end{aligned} \quad (4)$$

The Market Clearing Condition. The following conditions are needed for equilibrium:

(a) For the money market to be in equilibrium, it is necessary that $(Q_0)_{\text{Borrowed}} = - (Q_0)_{\text{Lent}}$, i.e. $Q'_0 = -Q_0 \Rightarrow Q'_0 + Q_0 = 0$ (5)

(b) For the goods market to be in equilibrium, it is necessary that $s =$ fractional shares owned by the taxable investor $= 1 - s' = 1 -$ fractional shares owned by the nontaxable investor, i.e., $s' + s = 1$. Furthermore, we need both $s' \geq 0$, and $s \geq 0$. (6)

(c) At time $t = 0$, it is required that:

$$c'_0 + Q'_0 + s'P_0 = w_0$$

$$c_0 + Q_0 + sP_0 = w_0$$

$$\therefore c'_0 + c_0 + P_0 = 2w_0 \quad (7)$$

(d) At time $t = 1$, it is required that:

$$c'_1 = Q'_0(1+i) + s'(d_1 + P_1)$$

$$c_1 = Q_0 [1 + i(1 - \tau)] + s[d_1(1 - \tau) + P_1(1 - g\tau) + \tau(P_0k(1 - g) + gP_0)]$$

$$\therefore c'_1 + c_1 = d_1 + P_1 + \tau\{s[-d_1 - gP_1 + P_0(k(1 - g) + g)] + Q'_0 i\} \quad (8)$$

The Solutions to the Model.

Subcase 1. First Corner Solution $s = 1$, i.e., taxable investors own all real estate. Price P_0 is given by equation (4) as:

$$P_0 = \gamma E_0 [d_1 [1 - \tau] + P_1 [1 - g\tau] + \tau [D_0 [1 - g] + gP_0]]$$

$$= \gamma E_0 \{d_1 [1 - \tau] + P_1 [1 - g\tau] + \tau [P_0 (k(1 - g) + g)]\} \text{ since } D_0 = kP_0$$

$$\therefore P_0 [1 - \gamma\tau(k(1 - g) + g)] = \gamma E_0 [d_1 (1 - \tau) + P_1 (1 - g\tau)]$$

$$P_0 = (P_0)_{\text{taxable}} = \frac{\gamma E_0 [d_1 [1 - \tau] + P_1 [1 - g\tau]]}{1 - \gamma\tau [k(1 - g) + g]}$$

Basic Condition for a Corner Solution:

$$P_0 = (P_0)_{\text{taxable}} > (P_0)_{\text{nontaxable}} = \gamma E_0 [d_1 + P_1]$$

$$\text{i.e., } P_0 = \gamma E_0 [d_1 [1 - \tau] + P_1 [1 - g\tau] + \tau [P_0 (k(1 - g) + g)]] > \gamma E_0 [d_1 + P_1]$$

$$\Rightarrow P_0 [k(1 - g) + g] > E_0 [d_1 + P_1 g]$$

$$\Rightarrow P_0 (k(1 - g)) > E_0 (d_1) + g E_0 (P_1 - P_0)$$

i.e., The tax benefit (net tax writeoff from depreciation) is greater than the tax liability of the net operating rental income plus the tax liability of the capital gain.

The Interest Rate:

The interest rate i is within an interval $[i_{\min}, i_{\max}]$

$$\text{i.e., } i \in [i_m, i_M]$$

The interest rate falls in this interval, because the lender (the non-taxable investor), prefers to lend at a rate above i_m . On the other hand, the borrower (the taxable investor) seeks loans at any rate below i_M .

i_{\max} is evaluated as follows:

$$\text{Equation (3) gives us: } 1 = \gamma E_0 [1 + i_M (1 - \tau)]$$

$$\Rightarrow P_0 = P_0 \gamma E_0 [1 + i_M (1 - \tau)]$$

Equation (4) gives us $P_0 = \gamma E_0[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-s)+g)]$

$$\therefore P_0 = P_0 \gamma E_0[1 + i_M(1-\tau)] = \gamma E_0[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-s)+g)]$$

$$\therefore P_0 E_0[1 + i_M(1-\tau)] = E_0[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-s)+g)]$$

$$\therefore E_0[1 + i_M(1-\tau)] = \frac{E_0[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g)]}{P_0}$$

$$\therefore 1 + i_M(1-\tau) = \frac{d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g)}{P_0}$$

$$\therefore i_M(1-\tau) = \frac{d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g) - P_0}{P_0}$$

$$\therefore i_{Max} = \frac{d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g) - P_0}{P_0(1-\tau)}$$

i_{min} is evaluated as follows:

Equation (3') gives us: $1 = \gamma E_0(1 + i_m)$

$$\Rightarrow P_0 = P_0 \gamma E_0(1 + i_m)$$

$$= \gamma E_0[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g)]$$

$$\therefore P_0 \gamma E_0(1 + i_m) = \gamma E_0[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g)]$$

$$\therefore E_0(1 + i_m) = \frac{E_0[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g)]}{P_0}$$

$$\therefore i_{min} = \frac{[d_1(1-\tau) + P_1(1-g\tau) + \tau P_0(k(1-g)+g)] - P_0}{P_0}$$

The optimal loan-to-value ratio is given as $(Q'_0)_{Max}$:

$$(Q'_0) = \text{Min. } w_0; \frac{\text{Min.}[d_1[1-\tau] + P_1[1-g\tau]] + \tau[P_0[1-g] + gP_0]}{[1+i(1-\tau)]}$$

(i) there is no initial capital constraint, and

(ii) the taxable investor is able to pay off the loan with interest even in the worst case.

The consumption levels are:

$$c'_0 = w_0 - (Q'_0)_{\text{Max}} ;$$

$$c_0 = w_0 + (Q'_0)_{\text{Max}} - P_0$$

$$c'_1 = (Q'_0)_{\text{Max}}(1+i) ;$$

$$c_1 = [d_1(1-\tau) + P_1(1-g\tau) + \tau (D_0(1-g) + gP_0)] - (Q'_0)_{\text{Max}}(1+i(1-\tau))$$

Subcase 2. Interior Solution: Both Investors Own Real Property. There is no interior solution, because Equation (3'), i.e., $\gamma E_0(1+i) = 1$, and Equation (3), i.e., $\gamma E_0(1+i(1-\tau)) = 1$ are not simultaneously true.

Subcase 3. Second Corner Solution: $s' = 1$ (i.e., the nontaxable investor owns all real estate).

$$\text{Equation (4')} \text{ gives us: } P_0 = \gamma E_0[d_1 + P_1]$$

The interest rate i is again within an interval $[i_{\text{Min}}, i_{\text{Max}}]$ i.e., $e [i_{\text{m}}, i_{\text{M}}]$

i_{Max} is evaluated as follows:

$$\text{From equation (3'), we have } 1 = \gamma E_0(1 + i_{\text{M}})$$

$$\Rightarrow P_0 = P_0 \gamma E_0(1 + i_{\text{M}})$$

$$\text{Now since } P_0 = \gamma E_0(d_1 + P_1)$$

$$P_0 = P_0 \gamma E_0(1 + i_{\text{M}})$$

$$\therefore P_0 = \gamma E_0(d_1 + P_1) = P_0 \gamma E_0(1 + i_{\text{M}})$$

$$\Rightarrow 1 + i_{\text{M}} = \frac{[d_1 + P_1]}{P_0}$$

$$i_{\text{Max}} = \frac{[d_1 + P_1 - P_0]}{P_0}$$

i_{min} is evaluated as follows:

$$\text{Equation (3) gives us } 1 = \gamma E_0[1 + i_{\text{min}}(1-\tau)] \Rightarrow P_0 = P_0 \gamma E_0[1 + i_{\text{min}}(1-\tau)]$$

Since $P_0 = \gamma E_0(d_1 + P_1)$

$P_0 = P_0 \gamma E_0 [1 + i_{\min}(1 - \tau)]$

$\therefore P_0 = \gamma E_0(d_1 + P_1) = P_0 \gamma E_0 [1 + i_{\min}(1 - \tau)]$

$$\therefore [1 + i_m(1 - \tau)] = \frac{[d_1 + P_1]}{P_0}$$

$$\Rightarrow i_m(1 - \tau) = \frac{[d_1 + P_1 - P_0]}{P_0}$$

$$i_m = \frac{[d_1 + P_1 - P_0]}{P_0(1 - \tau)}$$

Now the lender seeks to loan funds at a rate above i_m . On the other hand, the borrower seeks funds at a rate below i_m .

Since $i_{\min} > i_{\max} \Rightarrow$ this equilibrium does not exist.

\therefore We cannot have a second corner solution.

Thus the participating mortgage presented in this paper provides a quasi-equity alternative mechanism for nontaxable investors to enter the real estate market, for the solution (Subcase 1) illustrates that the lender receives the principal plus interest, which is a proportion of the Net Operating Income (NOI) and the capital appreciation of the property.

Conclusion

Equity participating mortgages are allowed by the Sharī'ah, for both the borrower and the lender assume the risk, there is no predetermined rate of return, and for several other reasons, as noted by Al-Qaradawi (1984).

Muslims in the United States can get a participating loan on their owner-occupied homes through Muslim Savings and Investments Inc. (MSI). This company will loan approximately 80 percent of a home's price. In lieu of interest payments, the borrower pays MSI a proportionate rent that takes into consideration the property's appreciation. The borrower can elect to buy out MSI in a scheduled period of fifteen years or less, again according to the appraised value of the property. Unfortunately, the rents paid to MSI are not tax deductible, as MSI fears that the IRS may regard equity participation as a joint venture and challenge any tax deductions by the owner. But there have been cases in which the

court has granted tax-deductible status to these mortgages: i.e., in *Hardman vs. U.S.* (1987), as reported by Levin (1990).¹⁰

MSI has a small portfolio (approximately \$4 - \$5 million). Its assets have not grown for several reasons: a) MSI does not sell mortgages on the secondary market as do most financial institutions; b) most Muslims are ignorant of alternatives to fixed interest investments; and c) declining real estate values have resulted in losses in MSI's real estate portfolio.

Other entities in the United States, such as Real Estate Investment Trusts (REITS), Real Estate Limited Partnerships (RELPS), and pension funds, invest in the type of participating mortgages described in this paper as well as in those based on some combination of fixed interest and equity participation. The ones with fixed interest (*ribā*), as already discussed, should be avoided. *Pension and Investment Age* (16 September 1991) reported that as of 30 June 1991, the fifty largest managers of tax-exempt assets had \$101.17 billion in real estate equity and \$14.86 billion in hybrid mortgages, which include participating and convertible mortgages.¹¹ Thus hybrid mortgages comprise approximately 14.7 percent of the equity real estate held by these institutions.

Participating mortgages are a recent innovation in real estate finance. However, the data above illustrates the demand for equity real estate investment and, as society becomes more aware of the opportunities in equity/profit sharing investment opportunities, equity participating mortgages of the kind discussed in this article will gain in popularity.

¹⁰Levin (1990) states that "In *Hardman versus the U.S.* (1987), the Ninth Circuit Court found that a shareholder was a creditor when vacant land was sold to a family corporation for acquisition cost and the corporation's promise to pay 1/3 of any net profit it derived from the property. The district court, focusing on the absence of an obligation to pay a principal sum with interest on a fixed maturity date held that the taxpayer was not a creditor but rather had received equity in the nature of a joint venture interest. However, the Court of Appeals for the Ninth Circuit reversed this decision. It considered 11 factors to be relevant in determining whether there was debt as a result of a genuine sale. A fixed maturity date was the only factor absent, and this was mitigated because repayment was tied to a fairly certain event—the sale of the property. All other factors supported debt status. Tax lawyers would be well advised to remember this case when structuring participating mortgages."

¹¹Convertible mortgages also involve fixed interest and are therefore not allowed by Islamic law.

Appendix A: Notational Glossary

Symbol	Interpretation	Variable Type
c_0	Consumption of taxable investor at $t=0$	Endogenous
c'_0	Consumption on nontaxable investor at $\tau=0$	Endogenous
c_1	Consumption of taxable investor at $t=1$	Endogenous
c'_1	Consumption on nontaxable investor at $\tau=1$	Endogenous
d_1	Net rents after operating expenses at period 1	Exogenous
D_0	Depreciation allowance	Exogenous
g	Capital gains rate i.e., 40% in old tax law	Exogenous
γ	Discount rate	Exogenous
i	Interest rate on money lent between $\tau=0$ and $\tau=1$	Endogenous
k	Depreciation constant i.e., rate of depreciation allowed i.e., $k = D_0 / P_0$	Exogenous
λ 's	Lagrange multipliers in the optimization problem for investors	Endogenous
P_0	Price of real estate at beginning i.e., $\tau=0$	Endogenous
P_1	Price of real estate at period 1	Exogenous
Q_0	Amount lent/borrowed at $\tau=0$ by taxable investor	Endogenous
Q'_0	Amount lent/borrowed at $\tau=0$ by nontaxable investor	Endogenous
s	Fractional ownership of real estate by taxable investor	Endogenous
s'	Fractional ownership of real estate by non-taxable investor	Endogenous
τ	Highest marginal tax rate	Exogenous
w_0	Endowment at $\tau=0$	Endogenous

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