A Zero Efficiency Loss Monopolist: An Islamic Perspective

Boualem Bendjilali and Farid B. Taher

Abstract

In an Islamic environment, the behavior of a single seller is different from that of a pure monopolist. His ultimate objective is not to maximize profit but to please Allah. Profit is only one of his motives. Therefore, he is expected to be ready to sacrifice part of his profits for the social good if and when the social priorities so require. This brief study seeks first to formulate this problem in its deterministic setting and to derive the optimally necessary conditions. Second, it examines the case of a family of utilities of the Cobb-Douglas form.

Introduction*

The term monopoly has commonly been used in microeconomic literature to describe the market condition of a single seller (the only supplier) who behaves in such a way as to maximize profits. As a profit maximizer, the firm produces less and charges higher prices than would be the case under perfect competition. Such behavior by the profit maximizing firm has several adverse impacts: first, it imposes a social-welfare loss (or efficiency loss) by producing a $P > MC$; second, it redistributes income from consumers to shareholders of the monopolist firm; third, it misallocates resources through the restriction of output. In addition, one may think of social costs of resources used by a monopolist firm for the protection and maintenance of its market power through nonprice competition practices, such as defensive advertising and non-necessary product differentiation.

In reality, the existence of such social costs calls for government

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interference through a set of alternative government pricing regulations. Unfortunately, however, government interference by itself destroys the market, and may generate an even greater efficiency loss to the society.

In an Islamic economy, monopoly as a market condition may prevail; in other words, the firm might be the only seller in the market. Nevertheless, the behavior of a Muslim firm is expected to differ from that of a non-Muslim monopolist because of the differences in their objectives. As a single seller, a Muslim producer is not expected to behave as a profit maximizer. The firm's decision-maker (manager, owner, shareholders) believes that restricting output in order to raise the price of a necessary good is a bad deed. According to the traditions of the Prophet (ṢAAS), it is reported that he said:

> "Whoever takes the advantage of a monopoly condition to raise the price to Muslims is a sinner,"

and that he also said:

> "monopoly of food in Makkah is atheism."

In Islam, the utility function of an individual depends on the welfare of other members of the society. This distinctive characteristic was best described by the Prophet (ṢAAS), when he said:

> "Muslims in their mercy and compassion are among themselves like one body; if one organ is sick other organs would show symptoms of sickness, too."

Muslims also believe that they should earn righteously, and that trade transactions should be arranged on a fair basis. For Allah says:

> "O ye who believe, eat not up your property among yourselves in vanities, but let there be amongst you traffic and trade by mutual goodwill, nor kill (or destroy) yourselves for Allah has been to you most Merciful." (4:29.)

Thus, a Muslim producer (single seller) is expected to be concerned about the welfare of the society, and therefore willing to partially sacrifice his or her own profits in order to avoid any loss in the welfare of the Muslim society in accordance with the preaching of the Messenger (ṢAAS), as he said:

> "Whoever is not concerned about Muslims' affairs is not a Muslim."

This behavioral approach of sacrificing profits for the benefit of society when it is needed has been mentioned by Siddiqi (10), who says:

> "In an Islamic society behaviour of all economic agents is expected to be socially oriented, ready to sacrifice profits for the social good, if and when the social priorities so require."

This paper is an attempt to formulate the objective function of a single seller in an Islamic economy. The paper will also assess the welfare implications of the expected outcomes, in comparison with those of the conventional monopolist case.

The paper is organized as follows: Section 1 will briefly present the welfare loss associated with the profit maximizing monopolist, and will discuss the effectiveness of government intervention. Section 2, the main core of this study, provides the formulation of the suggested Islamic model and the derivation of the necessary optimal conditions with application to the class of Cobb-Douglas utility functions. Section 3 is devoted to the conclusion.

1. Profit Maximizing Monopolist

Social Costs of Monopoly:

A pure monopolist in Figure (1) maximizes profit by producing (X_m) (where MC = MR), and charging (P_m). However, if the monopolist in this market is replaced by a large number of perfectly competitive firms, assuming no changes in cost functions, the aggregate marginal cost curve $\sum_{i=1}^{n} MC_i$ would coincide with the monopolist's (MC_m) curve and would become the market supply curve $\sum_{i=1}^{n} MC_i = MC_m = S$. Equilibrium would be attained at (X_c) and (P_c), which represent the society's efficient outcome because P_c = MC at this point.

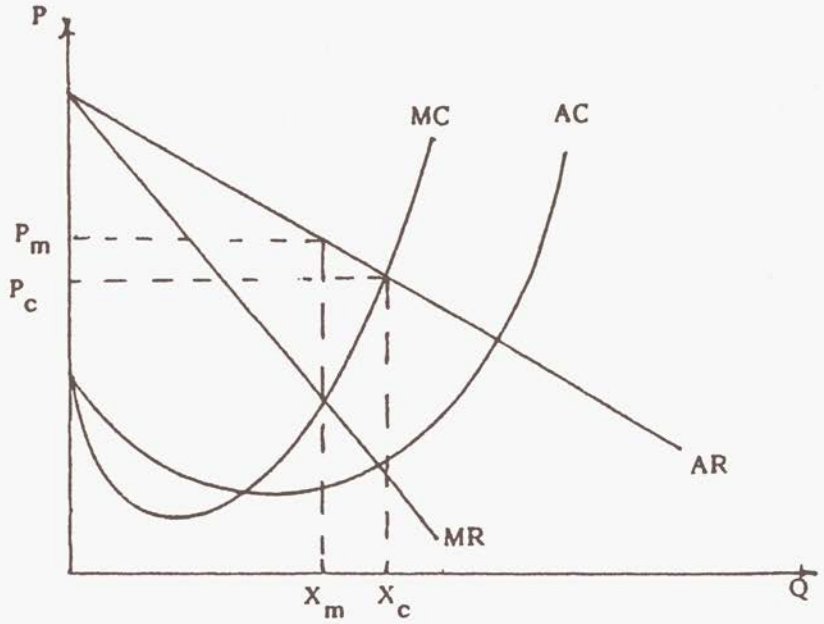

Fig. 1

The welfare cost of monopoly can be defined as the net gain in social welfare attainable by moving from (X_m) to (X_c). Net gains may be measured by aggregating social benefits (area under the demand curve) and social costs (area under the marginal cost curve) between (X_m) and (X_c), or the area Abc.

Government Interference

Governments interfere in monopoly situations to protect the public interest by imposing regulated prices. Such prices are assumed to induce production by the monopolist who tries to compensate for the reduction in price by increasing sales.

Unfortunately, however, regulating a monopoly may not be feasible at all times. Government officials frequently lack precise information concerning demand and cost functions of the monopolist firm. Apart from the information problem, regulation can only be enforced at some real cost. While eliminating the inefficiency associated with monopoly, government intervention would lead to inefficiencies in resource allocations by drawing resources away from otherwise efficient uses. That is why government interference has always been regarded as second best.

In the Islamic economy, a single seller is expected to be concerned about the social welfare and therefore to be willing to partially sacrifice his profits in order to attain efficiency and minimize social welfare loss. Such behavior would lead to the best solution under monopoly conditions. This point is the focus of the following section.

2. The Model

A. Assumptions

Let us assume a firm that produces a necessary good and assume that this firm is the only one in the market. Assume also that the firm is represented by its manager whose utility index depends on both profit and society's welfare. This dependence on profit and society's welfare in an Islamic environment comes from the fact that a Muslim economic agent has two main obligations: one toward self and family, and the second toward the society as a whole. Indeed, the Qur'an stresses the unity and caring among the individuals of the society. Moreover, the sayings of the Prophet (ṢAAS) point out this dependence between the individual Muslim (e.g., the firm's manager) and the society as a whole. If we let W and π denote the society's welfare gain as shown in Section 1 and the firm's profits respectively, the entrepreneur's utility U can then be written as:

$$(1)\quad U = U(\pi, w)$$

However, one knows that the welfare gain W is a function of the firm's output Q. Therefore, equation (1) can be written as:

$$(2)\quad U = U(\pi, Q)$$

since W is a function of Q.

Let Q_{max} and Q_e denote respectively the output levels corresponding to the maximum profit and the efficiency level of output, that is, to the quantity at which marginal cost equals price, as shown in Fig. 2.

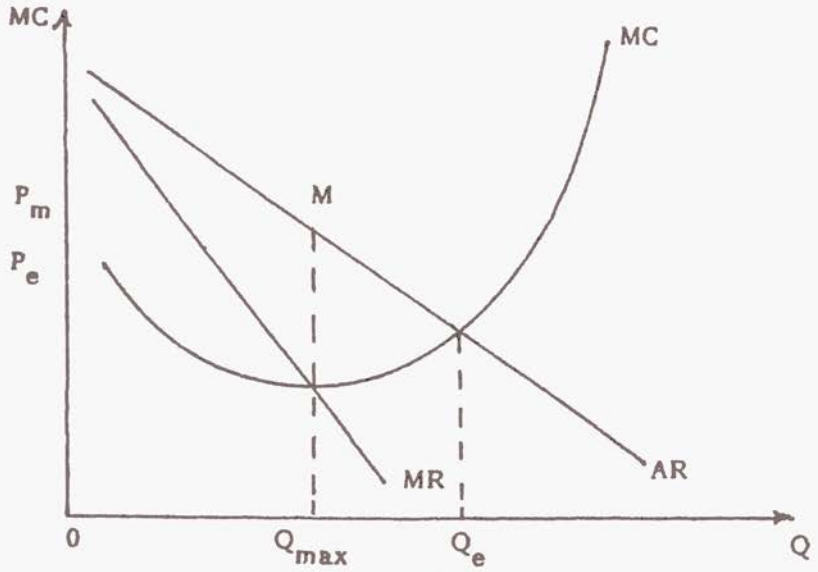

Fig. 2

In the case that the entrepreneur cares only about maximizing profit, he or she will produce at the level Q_{max} with a corresponding society's welfare loss equal to W_o, where W_o has been defined in the first section. However, in the case that the entrepreneur cares about the society's welfare in addition to profit, the output produced will be between Q_{max} and Q_e. Indeed, the more the entrepreneur cares about the society's welfare the more he or she moves from the point M toward the efficiency point E. That is, the quantity produced moves from Q_{max} toward the level of output Q_e. At the efficiency point E, the society's welfare loss will be equal to zero.

The following properties of the entrepreneur's utility function U are assumed.

H.1 The marginal utilities of U with respect to both arguments are positive. In other words $\frac{\partial U}{\partial \Pi} > 0$ and $\frac{\partial U}{\partial Q} > 0$.

H.2 The second partial derivatives $\frac{\partial^2 U}{\partial \Pi^2}$ and $\frac{\partial^2 U}{\partial Q^2}$ are negative.

H.3 The cross partial derivatives $\frac{\partial^2 U}{\partial Q \partial \Pi}$ and $\frac{\partial^2 U}{\partial \Pi \partial Q}$ are positive.

Assumptions H.1 and H.2 together mean that for a fixed level of output (for a fixed level of profit), the entrepreneur's utility function increases with a decreasing rate as profit (as output Q) increases. Assumption H.3 indicates

that for a fixed level of profit (for fixed level of output), the entrepreneur's utility curve will rotate around the origins as a pivot as indicated in Fig. 3, as output increases (as profit increases).

In addition to the first three assumptions, we assume a given total cost function TC(Q) and a given total revenue function TR(Q), Accordingly, we have a given profit function (Q). The general objective function of the entrepreneur is to maximize the utility function subject to the profit-output transformation curve.

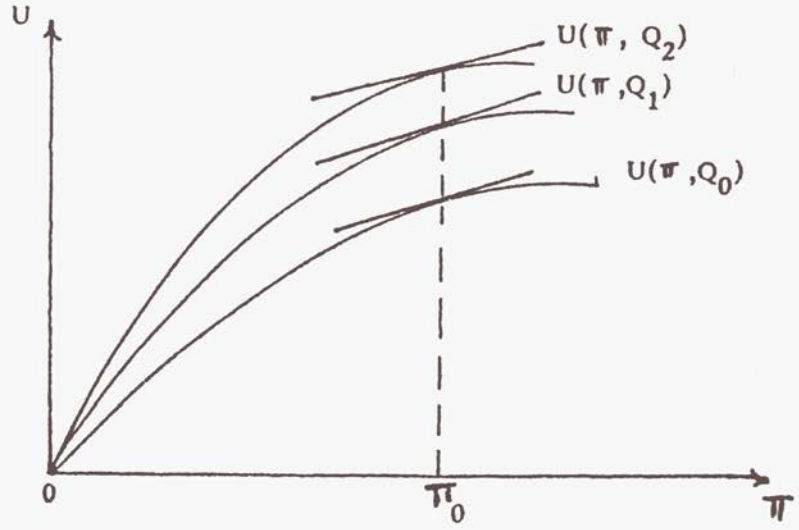

Fig. 3

Where $Q_0 < Q_1 < Q_2$.

The profit-output transformation curve can be written as:

(3) $\quad = TR\ (Q) - TC(Q)$

where $TR.Q = p(Q).Q$

B. Specific Formulation of the Decision Problem and Optimal Conditions

The utility maximization as perceived by the entrepreneur is:

(4) Max $U = \text{Max } U\ (\pi, Q)$ subject to

$$\pi = TR\ (Q) - TC\ (Q)$$

The current-value Lagrangian is equal to:

$$(5)\quad L = U(\pi, Q) - \lambda [\pi - TR(Q) + TC(Q)]$$

The necessary conditions for maximizing the entrepreneur utility function are:

$$(6)\quad \frac{\partial L}{\partial \Pi} = U_{\Pi} - \lambda = 0$$

$$(7)\quad \frac{\partial L}{\partial Q} = U_Q - \lambda(MC - MR) = 0$$

$$(8)\quad \frac{\partial L}{\partial \lambda} = TR(Q) - TC(Q) - \Pi = 0$$

From equations (6) and (7) we get:

$$(9)\quad \frac{U_Q}{U_{\Pi}} = MC - MR \quad ; \qquad \lambda > 0$$

The left-hand side of equation (9) is the marginal rate of substitution between the level of output Q and the level of profit that is the $MRS_{Q\pi}$. Namely, equation (9) can then be rewritten as:

$$(10)\quad MRS_{Q\pi} = MC - MR$$

and

$$(11)\quad \pi = TR(Q) - TC(Q)$$

Equations (10) and (11) represent the necessary conditions for maximization. We notice from equation (10) that in the case of $MRS_{Q,\pi} = 0$, this will imply that MC = MR and therefore we are in the case of a pure monopolist. On the other hand, in the case where $Q = Q_e$ we have MC = p and hence the necessary condition (10) becomes:

$$MRS_{Q,\pi} = p - p - p'Q = p'Q_e$$

That is:

$$(12)\quad MRS_{Q,\pi} = - p'(Q_e)\, Q_e$$

See Fig. 4 and Fig. 5 for more geometric clarifications. Therefore, in order for the firm to produce a level of output between Q_{max} and Q_e, the necessary condition becomes:

$$(13)\quad 0 < MRS_{Q,\pi} < - p'(Q_e)\, Q_e$$

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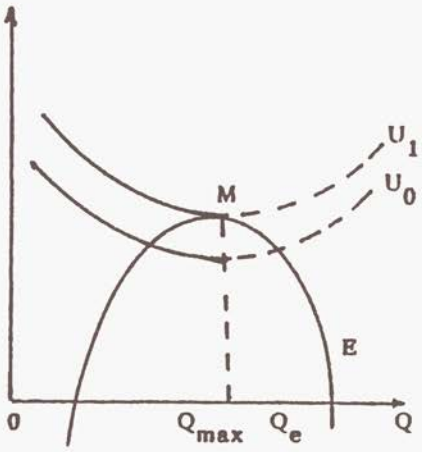

Fig. 4

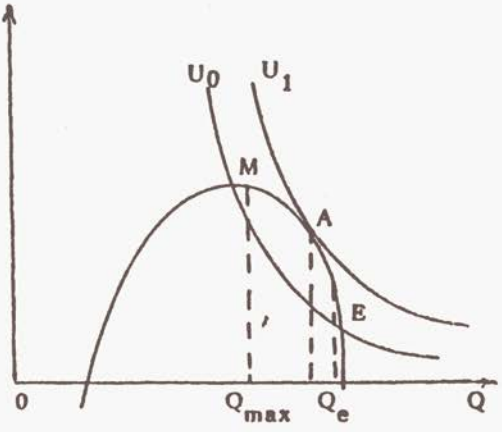

Fig. 5

Next, we would like to check the second order for maximization. By differentiating the first order conditions with respect to λ and Q we get:

$$\Delta = \begin{vmatrix} 0 & -1 & MR-MC \\ -1 & U_{\Pi\Pi} & U_{\Pi Q} \\ MR-MC & U_{Q,\Pi} & U_{QQ}-\lambda\,(MC'-MR') \end{vmatrix}$$

where MC' and MR' denote the derivatives of the marginal cost and marginal revenue with respect to output Q. The second order conditions for maximization must satisfy $\Delta_1 < 0$ and $\Delta_2 >$ where:

$$\Delta_1 = \begin{vmatrix} 0 & -1 \\ -1 & U_{\Pi\Pi} \end{vmatrix} = -1 < 0$$

and after a simple computation we get:

$$\Delta_2 = \lambda(MC'-MR') - U_{QQ}-U_{Q\pi} (MR-MC) + (MR-MC) \Big\{ -U_{\pi Q}- U_{\pi\pi} (MR-MC) \Big\}$$

$$2 = U_{\pi} (MC'-MR') - U_{QQ} - 2(MR-MC) U_{\pi Q} - U_{\pi\pi} (MR-MC)^2$$

Using assumptions H1-H3 and the properties of the marginal revenue and marginal cost, we deduce that Δ_2 is positive and hence the second order conditions are satisfied.

C. Application to the Cobb-Douglas Class of Utilities

Let us next consider the class of utilities represented in the form of Cobb-Douglas. That is,

insert equation here

We notice first that $MRS_{q,\pi} \neq 0$ since $U_q \neq 0$, since $\beta > 0$ and π and q are strictly positive. Moreover, U is strictly quasi concave. This is easily shown by looking at the following minor determinants.

$$(14) \quad \begin{vmatrix} 0 & \alpha \frac{U}{\pi} & \beta \frac{U}{q} \\ \frac{\alpha U}{\pi} & \alpha(\alpha-1)\frac{U}{\pi^2} & \frac{\alpha\beta U}{\pi q} \\ \beta \frac{U}{q} & \frac{\alpha\beta U}{\pi q} & \frac{\beta(\beta-1)U}{q^2} \end{vmatrix} = B$$

Then the minors

$$(15) \quad B_1 = -\frac{\alpha^2 U^2}{\pi^2} < 0 \qquad \text{and}$$

$$(16) \quad B_2 = B = \frac{\beta\alpha U^3(\alpha+\beta)}{\pi^2 q^2} > 0$$

We are interested in what follows in looking at the restrictions that must be imposed α and β in order to get the level of output and level of profit

that maximize the utility function of the entrepreneur, other than the case of a pure monopolist. Given the total revenue and total cost function, let the profit function be given by:

$$\pi = pq - TC(q)$$

The problem becomes:

$$\max U = \pi^{\alpha} q^{\beta} \qquad \text{subject to}$$

$$\pi = pq - TC(q)$$

The current value Lagrangian becomes:

$$(17)\quad L = \pi^{\alpha} q^{\beta} - \lambda [\pi - pq + TC(q)]$$

A simple computation gives the necessary conditions:

$$(18)\quad MRS_{q,\pi} = \frac{\beta \pi}{\alpha q} = MC - MR$$

$$(19)\quad \pi = pq - TC(q)$$

The second order conditions are derived from the following minors of the following Hessian.

Let

$$H = \begin{vmatrix} 0 & -1 & MR - MC \\ -1 & \frac{\alpha(\alpha-1)U}{\pi^2} & \frac{\alpha\beta U}{\pi q} \\ MR-MC & \frac{\alpha\beta U}{\pi q} & \frac{\beta(\beta-1)}{q^2} U - \lambda (MC'-MR') \end{vmatrix}$$

We know from the first order conditions that we must have the following minors H_1 and H_2 as follows:

$$H_1 < 0 \text{ and } H_2 > 0$$

we get

$$H_1 = \begin{vmatrix} 0 & -1 \\ -1 & \frac{\alpha(\alpha-1)U}{\Pi^2} \end{vmatrix} = -1 < 0$$

Moreover, using equation (18) we get by a simple computation

$$H_2 = U\left\{\frac{-4\beta^2}{q^2} + \frac{\beta}{q^2} + \frac{\beta 2}{\alpha q^2}\right\} + \lambda (MC' - MR')$$

we know that

$$MC - MR = \frac{\beta\Pi}{\alpha q} \qquad \text{and} \qquad \lambda = \frac{\alpha U}{\Pi}$$

Hence

$$(20)\quad MC' - MR' = \frac{-\beta\Pi}{\alpha q^2}$$

Using this last information H_2 becomes

$$(21)\quad H_2 = U\left\{\frac{\beta^2}{\alpha q^2} - \frac{4\beta^2}{q^2}\right\} = \frac{U\beta^2}{\alpha q^2}(1-4\alpha)$$

The second order conditions impose that H_2 must be positive and hence, from equation (21) we must have (1-4a) > 0, that is,

$$(21)\quad 0 < \alpha < 1/4$$

Equation (21) gives the optimal conditions that permit production between Q_{max} and Q_e. In other words, the elasticity of the utility with respect to profit is restricted to be between 0 and 1/4 for the class of utilities of the Cobb-Douglas form, in order for the entrepreneur to maximize utility and produce an output between the quantity Q_{max} corresponding to the level of output produced in the case of a pure monopolist and the efficient level of output.

3. Conclusion

In an Islamic economy, monopoly (single seller) as a market condition may prevail, however, the behavior of a Muslim is different from that of a non-Muslim, simply because their objectives and motives are different. A Muslim single seller is expected to sacrifice part of his profit (according to the degree of faith he possesses) for the benefit of the welfare of the poor and the needy of the society that is, for the benefit of the social welfare of the society. This paper has been an attempt toward achieving this goal. It formulated the objective function of a single seller in an Islamic economy. It also derived the necessary and sufficient optimality conditions for maximization. The last section examined the case of a family of utilities of the Cobb-Douglas form. This study has examined this problem in its deterministic setting as well as in its static form. Two potentially interesting issues that we do not address here are the possibility of examining this problem in its dynamic form and the other in its probabilistic approach. These two issues will be part of our future research.

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